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Relativistic effects, QCD mixing angles and $N\to N\gamma$ and $\Delta\to N\gamma$ transition form factors

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Abstract

It is shown that relativistic effects, considered in the framework of a relativistic quark model constructed in infinite momentum frame, improve the agreement between theory and experiment for $\Delta\to N\gamma$ transition. They enlarge the magnitudes of the amplitudes $A^p_{1/2}$ and $A^p_{3/2}$ and suppress with increasing Q^2 the magnetic form factor of $\Delta\to N\gamma$ transition in comparison with proton magnetic form factor. The additional inclusion of not large QCD-inspired mixings of multiplet $(56',0^+)$ into the N and the Δ improves further the agreement with experiment for this form factor and permits to describe its Q^2-dependence at $Q^2 < 3$ GeV^2. Predominantly due to the relativistic effects non-zero values for electric and Coulombic form factors of $\Delta\to N\gamma$ transition are obtained. It is predicted that the electric form factor is positive at $Q^2 < 0.4$ GeV^2 and changes its sign with increasing Q^2, so the magnitude of helicity asymmetry should be lower than 0.5 at $Q^2 > 0.4$ GeV^2.

1. Introduction

It is known that the predictions of the nonrelativistic quark model for $\Delta(1232)\to N\gamma$ transition are unsatisfactory. The predicted values of the amplitudes of this transition are significantly lower than experimental data, and the inclusion of QCD mixing effects does not remove this descreapancy [1,2,3]. Further, the nonrelativistic quark model predicts the same Q^2-dependence for the magnetic form factors of $N\to N\gamma$ and $\Delta\to N\gamma$ transitions, whereas the experiment shows that with increasing Q^2 the $\Delta\to N\gamma$ transition form factor falls more rapidly than nucleon ones [4,5]. Such a failure of the nonrelativistic quark model in the description of $\Delta\to N\gamma$ transition cannot be considered as surprising because the calculations are highly relativistic here due to the large momentum transfer, and the nonrelativistic approximation is not adequate to describe them. Various relativistic correction terms to the nonrelativistic approximation for radiative transitions of nucleon and nucleon resonances are studied in Refs. [6-8] where it is shown that relativistic effects are significant. However the obtained results cannot give a complete picture of the relativistic effects as quarks in the considered processes are relativistic and it is neccessary to take into account the whole power series in quark velocity.

In this work we present our results on joint description of $N\to N\gamma$ and $\Delta\to N\gamma$ transitions obtained in the framework of relativistic quark model (RQM) of Ref. [9], which was successfully used to describe the static properties and form factors of the nucleon

[9,10]. This model is constructed in infinite momentum frame (IMF) using of which (as well as of the light-front dynamics in the models of Refs. [11-16]) have advantages when we study interactions of bound states of relativistic constituents:

i) RQM of Ref.[9] is constructed for radiative (or weak) transitions of hadrons $A\to B+\gamma$ using the formalism of old-fashioned perturbation theory in the IMF where the initial hadron moves along the z axis with momentum $P\to\infty$ and the longitudinal components of photon momentum are:

$$q_0 \simeq -q_z = \frac{m_A^2 - m_B^2 + q^2}{4P} \quad , \quad q^2 = - q_\perp^2 \equiv -Q^2 . \qquad (1)$$

In this frame triangular diagrams containing creation or annigilation of quark-antiqurk pairs in the vertex of interaction with photon are suppressed; hence, space-time picture of the process corresponds to the nonrelativistic quantum mechanics and only diagrams with vertices which have wave function interpretation remain.

ii) In the processes of hadron transitions with high momentum transfer it is neccessary to take into account the motion of the hadrons. In our choosen IMF it is the motion of final hadron in transverse direction. In Ref. [9] a relativistic-covariant form of spin-orbital parts of hadron wave functions are found and using them it is shown that in IMF hadron wave functions differ from the wave functions of quarks in their c.m.s. only by the quark spin rotations given by Melosh matrices. Herewith the transverse momentum of final hadron enters in the definite form into the c.m.s wave function of final guarks and into corresponding Melosh matrices. So, in the IMF we have a formalism which allows to take into account the momentum trasfer between hadrons.

In this paper we will investigate the role of the relativistic effects and QCD-inspired admixtures of higher exitation states in the description of the $N \to N\gamma$ and $\Delta \to N\gamma$ transition form factors. In particular, we will be interesting, whether these effects can avoid the descreapancies between the nonrelativistic quark model predictions and the experiment for the $\Delta \to N\gamma$ transition. To investigate the role of QCD-inspired mixings we will use the results of the "soft QCD-model" of Refs.[17-19] which includes many features of QCD [20] and describe successfully the mass spectrum of practically all light mesons and baryons. This description is made in terms of nonrelativistic spin-orbital wave functions, while radial parts of wave functions correspond to the relativistic quarks and mean momenta of quarks obtained in [17-19] are of the order of quark masses. In the "soft QCD model" it is found that for the nucleon and the Δ-isobar the main admixtures of higher configuration states come from multiplets $[56',0^+]$ and $[70,0^+]$:

$$|N\rangle = a_N |N[56,0^+]\rangle + b_N |N[56',0^+]\rangle + c_N |N[70,0^+]\rangle \quad . \tag{2.1}$$

$$|\Delta\rangle = a_\Delta |\Delta[56,0^+]\rangle + b_\Delta |\Delta[56',0^+]\rangle \quad , \tag{2.2}$$

where the coefficients are determined by hyperfine interaction generating by one-gluon exchange and consisting of the Fermi contact term and a tensor term. Their values are correlated with each other and have definite signs:

$$b_N \approx c_N \approx - b_\Delta < 0 \quad . \tag{2.3}$$

Therefore we will investigate the role of the admixtures of multiplets $[56',0^+]$ and $[70,0^+]$ in N and Δ taken with the

coefficients which obey the relation (2.3).

2 Basic formulae

As the initial formulae of RQM are given repeatedly in our previous papers (see, for example, Refs. [9,10]) we start immediately with the matrix element of the electromagnetic current for radiative transition $A(P,\lambda) \to B(P',\lambda') + \gamma(q)$ (in parentheses the momenta and the helicities of the particles are given) which in IMF has the following form:

$$\langle B(P',\lambda')|J_\mu|A(P,\lambda)\rangle|_{P\to\infty} = 3eQ_a \int d\Gamma \Psi_B^+(p'_a,p'_b,p'_c)\Gamma_\mu \Psi_A(p_a,p_b,p_c) . \tag{3}$$

Here it is supposed that the photon interacts with quark a (the quarks in hadrons are denoted by a,b,c), Q_a is the charge of this quark (in units of e) and p_i and p_i' (i=a,b,c) are the guark momenta which we parametrize in the IMF in the following way:

$$p_i = x_i P + K_{i\perp} \; , \qquad p_i' = x_i P' + K_{i\perp} \; , \qquad P K_{i\perp} = P' K'_{i\perp} = 0, \; i=a,b,c,$$

$$\begin{array}{lll} x_1 = 1-\eta, & K_{a\perp} = -K_\perp , & K'_{a\perp} = -K'_\perp \; , \\ x_2 = (1-\zeta)\eta \; , & K_{b\perp} = k_\perp + (1-\zeta)K_\perp , & K_{b\perp} = k_\perp + (1-\zeta)K'_\perp \; . \\ x_3 = \zeta\eta \; , & K_{c\perp} = -k_\perp + \zeta K_\perp \; , & K_{c\perp} = -k_\perp + \zeta K'_\perp , \end{array} \tag{4}$$

$K'_\perp = K_\perp + \eta q_\perp$. The phase space folume $d\Gamma$ in (3) is equal to

$$d\Gamma = \frac{dk_\perp dK_\perp d\zeta d\eta}{(2\pi)^6 4\zeta(1-\zeta)\eta(1-\eta)} \quad . \tag{5}$$

Γ_μ corresponds to the vertex of the quark-photon interaction $\Gamma_\mu = \bar{u}(p_a')\gamma_\mu u(p_a)/x_a$ which has the following components:

$$\Gamma_0 = \Gamma_z = 2P \; , \; \Gamma_x = -(2K_x + q_x)/(1-\eta) \; , \; \Gamma_y = -(2K_y + i\sigma_z^{(a)} q_x)/(1-\eta) . \tag{6}$$

Here we have assumed for convenience that the vector $q_\perp$ is directed along the X axis. In order to define the wave functions of the systems of initial and final quarks it is convenient to introduce the 4-momenta of the quarks in their c.m.s. $K_i(K_{i\perp}, K_{iz}, \omega_i)$ and $K'_i(K_{i\perp}, K'_{iz}, \omega'_i)$ $(i=a,b,c)$:

$$K_{iz} + \omega_i = M_0 x_i , \qquad \omega_i = \sqrt{K_i^2 + m^2} , \qquad M_0 = \sum_i \omega_i , \tag{7}$$

$$K'_{iz} + \omega'_i = M'_0 x_i , \qquad \omega'_i = \sqrt{K_i'^2 + m^2} , \qquad M'_0 = \sum_i \omega'_i ,$$

where m is the mass of quarks a,b,c . M_0 and M'_0 are invariant masses of the initial and final quarks:

$$M_0^2 = \sum_i (K_{i\perp}^2 + m^2)/x_i , \qquad M_0'^2 = \sum_i (K_{i\perp}'^2 + m^2)/x_i . \tag{8}$$

According to the results of Ref [9] the vertex function of hadron transition to quarks in the IMF is related to the wave function of quarks in their c.m.s by quark spin rotations:

$$\Psi_{A,\lambda} = U^+(K_a)\, U^+(K_b)\, U^+(K_c)\, \Psi_{A,\lambda}^{c.m.s}(K_a, K_b, K_c) , \tag{9}$$

where $U(K_i)$ are Melosh matrices :

$$U(K_i) = \frac{m_i + i\varepsilon_{lm}\sigma_l K_{lm}}{\left[m_i^2 + K_{i\perp}^2 \right]^{1/2}} , \qquad m_i \equiv m + M_0 x_i . \tag{10}$$

From (3), (6) and (9) it is seen that the final results will be expressed in terms of matrices $U(K'_i)\, U^+(K_i)$ $(i=a,b,c)$ and $U(K'_a)\, \sigma_z^{(a)} U^+(K_a)$ taken between wave functions of initial and final quarks in their c.m.s . By this reason it is convenient to write down these matrices directly:

$$U(K'_i)\, U^+(K_i) \equiv \begin{pmatrix} a_{11}^{(i)} & a_{12}^{(i)} \\ a_{21}^{(i)} & a_{22}^{(i)} \end{pmatrix} / D_i , \qquad i=a,b,c , \tag{11}$$

$$U(K'_a)\sigma_z^{(a)} U^+(K_a) \equiv \begin{pmatrix} \alpha_{12} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} / D_a , \tag{12}$$

where

$$a_{11(22)}^{(i)} = m_i m'_i + K_{ix}K'_{ix} + K_{iy}^2 \pm i(K'_{ix} - K_{ix})\, K_{iy} , \tag{13.1}$$

$$a_{12(21)}^{(i)} = \pm(m'_i K_{ix} - m_i K'_{ix}) + i x_i (M_0 - M'_0)\, K_{iy} . \tag{13.2}$$

$$\alpha_{11(22)} = \pm(m_i m'_i - K_{ax}K'_{ax} - K_{ay}^2) + i(K'_{ax} - K_{ax})K_{ay} , \tag{13.3}$$

$$\alpha_{12(21)} = -(K_x m'_a + K'_x m_a) \mp i(m_a + m'_a)K_{ay} , \tag{13.4}$$

$$D_i = \{(m_i^2 + K_{i\perp}^2)(m_i'^2 + K_{i\perp}'^2)\}^{1/2} , \tag{13.5}$$

In order to find the relations between matrix elements (3) and form factors of radiative transitions under consideration let us write covariant expressions for their currents:

$$\langle 1/2^+ \lambda' | J_\mu | 1/2^+ \lambda \rangle = e\bar{u}_{\lambda'}(P')\{\tilde{F}_1(q^2)(q^2\gamma_\mu - q_\mu \not{q}) - F_2(q^2)\sigma_{\mu\nu}q^\nu\}\, u_\lambda(P), \tag{14}$$

$$\langle 1/2^+ \lambda' | J_\mu | 3/2^+ \lambda \rangle = e\bar{u}_{\lambda'}(P')\gamma_5 (H_{\beta\mu}^{(1)} + H_{\beta\mu}^{(2)} + H_{\beta\mu}^{(3)})u_\lambda^\beta(P), \tag{15}$$

where

$$H_{\beta\mu}^{(1)} = i\tilde{G}_1(q^2)\, \varepsilon_{\beta\mu\nu\sigma}q^\nu\gamma^\sigma , \quad H_{\beta\mu}^{(2)} = \tilde{G}_2(q^2)q_\beta\sigma_{\mu\nu}q^\nu , \tag{16}$$

$$H_{\beta\mu}^{(3)} = \tilde{G}_3(q^2)\,(q_\beta q_\nu - q^2 g_{\beta\mu}) , \quad \sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2 ,$$

and $u^\beta(P)$ is the Rarita-Schwinger bispinor. For all bispinors we use the normalization $\bar{u}u = 2m$.

The invariant amplitudes of Eqs.(14-16) are related to the matrix elements (3) in the following way:

$$\frac{1}{2P} \langle N, \lambda'=1/2 \,|\, J_0 \,|\, N, \lambda=1/2 \rangle \big|_{P\to\infty} = q^2\, \tilde{F}_1(q^2) \equiv F_1(q^2) , \tag{17.1}$$

$$\frac{1}{2P} \langle N, \lambda'=-1/2 \,|\, J_0 \,|\, N, \lambda=1/2 \rangle \big|_{P\to\infty} = -F_2(q^2)q_x , \tag{17.2}$$

$$\frac{1}{2P} \langle N, \lambda'=1/2 \,|\, J_0 \,|\, \Delta, \lambda=3/2 \rangle \big|_{P\to\infty} = -\frac{q_x}{\sqrt{2}}\, \tilde{G}_1(q^2) , \tag{17.3}$$

$$\frac{1}{2P} \langle N, \lambda'=-1/2 \,|\, J_0 \,|\, \Delta, \lambda=3/2 \rangle \big|_{P\to\infty} = \frac{q_x^2}{\sqrt{2}}\, \tilde{G}_2(q^2) , \tag{17.4}$$

$$\langle N,\lambda'=-1/2 \mid J_x + iJ_y \mid \Delta,\lambda=3/2 \rangle|_{P\to\infty} = \frac{q_x^3}{\sqrt{2}} \tilde{G}_3(q^2) \quad . \tag{17.5}$$

For $N\to N\gamma$ transition the amplitudes $F_1 \equiv q^2\tilde{F}_1$ and F_2 are the Pauli form factors which are related to the Sachs form factors by:

$$G_E^N(q^2)=F_1(q^2) + \frac{q^2}{2m_N} F_2(q^2) \ , \tag{18.1}$$

$$G_M^N(q^2)=F_1(q^2) + 2m_N F_2(q^2) \tag{18.2}$$

For $\Delta\to N\gamma$ transition it is convenient to introduce helicity amplitudes h_1, h_2, h_3 defined in Ref. [21] and related to the amplitudes $\tilde{G}_1(q^2)$, $\tilde{G}_2(q^2)$ and $\tilde{G}_3(q^2)$ by:

$$h_1 = 4m_\Delta[\tilde{G}_1+(m_\Delta+m_N)\tilde{G}_2]+2(m_\Delta^2-m_N^2+q^2)\tilde{G}_3 \ , \tag{19.1}$$

$$h_2=-2\{(m_\Delta+m_N)\tilde{G}_1+q^2(\tilde{G}_2+\tilde{G}_3)\} \quad , \tag{19.2}$$

$$h_3=2/m_\Delta\{[q^2-m_N(m_\Delta+m_N)]\tilde{G}_1+[(m_\Delta^2-m_N^2)(m_\Delta+m_N)+m_N q^2]\tilde{G}_2+m_\Delta q^2\tilde{G}_3\} \ . \tag{19.3}$$

The Sachs form factors and helicity amplitudes of $\Delta\to N\gamma$ transition are connected with these amplitudes in the following way:

$$G_M^\Delta = -\frac{m_N}{6(m_\Delta+m_N)} (3h_2+h_3) \ , \tag{20.1}$$

$$G_E^\Delta = \frac{m_N}{6(m_\Delta+m_N)} (h_3-h_2) \ , \tag{20.2}$$

$$G_C^\Delta = \frac{m_N}{3(m_\Delta+m_N)} h_1 \ . \tag{20.3}$$

$$A_{1/2} = \frac{e}{4\sqrt{3}} h_3 \left[\frac{(m_\Delta-m_N)^2-q^2}{(m_\Delta^2-m_N^2)m_N}\right]^{1/2} . \tag{20.4}$$

$$A_{3/2} = \frac{e}{4} h_2 \left[\frac{(m_\Delta-m_N)^2-q^2}{(m_\Delta^2-m_N^2)m_N}\right]^{1/2} . \tag{20.5}$$

Let us remind that for multipole amplitudes of the $\gamma N\to\pi N$ reaction we have

$$\frac{E_{1+}}{M_{1+}} = -\frac{G_E}{G_M} \ , \quad \frac{S_{1+}}{M_{1+}} = -\frac{G_C}{G_M}\frac{q_{cms}}{2m_N} \ . \tag{21}$$

So, we have related all measured on the experiment amplitudes of $N\to N\gamma$ and $\Delta\to N\gamma$ transitions to the matrix elements (3) which we calculate in our approach in the IMF. The concrete form of the matrix elements for transitions between members of multiplets $[56,0^+]$, $[56',0^+]$ and $[70,0^+]$ are given in Appendix.

In this paper we will give also our results on nucleon axial vector form factor which in the considered approach is determined by :

$$g_A(q^2) = 3 \int d\Gamma\, \Psi^+_{n,1/2}(K_a',K_b',K_c')\, \sigma_z^{(a)}\tau_-^{(a)}\Psi_{p,1/2}(K_a,K_b,K_c) \tag{22}$$

where τ_- is the matrix realizing the transition $d\to u$. The concrete form of (22) for transitions between various multiplets is given in Appendix.

3 Results

3.1 The role of relativistic effects

In order to investigate the role of relativistic effects in the description of the $N\to N\gamma$ and $\Delta\to N\gamma$ form factors let us start with the case when admixtures of higher exitation states in N and Δ are absent.

The radial part of the wave functions we take in the form

$$\Psi_r = \exp(-M_0^2/6\alpha^2) \tag{23}$$

which is very close to the harmonic oscillator wave function and coincides with it in the nonrelativistic limit. Paramter α characterizes the value of quark momenta in N and Δ. In Refs [10,12] it was shown that the wave function (23), in spite of exponential form, give power-like behaviour of form factors in the region of not too large Q^2. This fact can be easily shown using a simplified formula for $F_1^p(Q^2)$ which can be obtained from (17.1) and (A.1) by ignoring of the Melosh matrices :

$$F_1^p(Q^2) \sim \int \exp\left[-(M_0^2 + M_0'^2)/8\alpha^2\right] \sim \int \exp\left\{ -\frac{Q^2\eta}{12\,\alpha^2(1-\eta)} - \frac{m^2}{3\alpha^2}\left[\frac{1}{\zeta(1-\zeta)\eta} + \frac{1}{1-\eta}\right]\right\}\eta\, d\eta\, d\zeta . \tag{24}$$

For a relativistic system with $\alpha^2/m^2 \gg 1$ and for large Q^2 taken in the interval

$$12\alpha^2 < Q^2 < 9\,\alpha^4/m^2 \tag{25}$$

the main contribution to the inte -s from the region $\eta < 12\alpha^2/Q^2$, where the exponent depends weakly on Q^2, and the behaviour of the form factor is determined mainly by the phase space volume; whence it follows $F_1^p(Q^2) \sim \alpha^4/Q^4$ At larger values of Q^2 the main contribition comes from the region $\eta < 4m/Q$,and power-like fall transforms gradually into a behaviour of the form of $m^2/Q^2 \exp(-2mQ/3\alpha^2)$. The exact formulae differ from (24) by the pre-exponential factors which are different for different form factors. The influence of these factors can be evaluated in the limit $\zeta = 1/2$, $K_{i\perp} = 0$, $(i = a,b,c)$ if we preserve the variable η which is crucial for Q^2-dependence of form factors. Using the expressions (17.1)-(17.3), (22) and the formulae of Appendix we obtain in this limit:

$$F(Q^2) = N^2 \int F \exp[-(M_0^2 + M_0'^2)/8\alpha^2]\, d\Gamma \quad . \tag{26}$$

where

$$F = 2Q_u + Q_d \quad \text{for } F_1^p(Q^2) , \tag{27.1}$$

$$F = \eta/2m\,(Q_u - Q_d) \text{ for } F_2^p(Q^2) , \tag{27.2}$$

$$F = \sqrt{3}\,\eta/2m\,(Q_u - Q_d) \text{ for } \tilde{G}_1(Q^2) , \tag{27.3}$$

$$F = 5/3 \quad \text{for } g_A(Q^2) . \tag{27.4}$$

At large Q^2 from the interval (25) in the main region of integration the variable η behaves as $\eta \sim 1/Q^2$, so we find from (26) and (27) that form factors $F_2^p(Q^2)$, $F_2^n(Q^2)$ and $\tilde{G}_1(Q^2)$ fall more rapidly with increasing Q^2 than $F_1^p(Q^2)$ and $g_A(Q^2)$. So, in our approach we have simple explanation of the experimentally observed fact, that the form factor $g_A(Q^2)$ falls more slowly with increasing Q^2 than the proton magnetic form factor $G_m^p(Q^2) = F_1^p(Q^2) + 2m_N F_2^p(Q^2)$, and the last one falls more slowly than form factor $G_M^\Delta(Q^2)$ which is determined mainly by $\tilde{G}_1(Q^2)$. As it will be shown below these qualitative conclusions are confirmed by exact calculations.

We find parameters m and α of our model from the requirement of a best description of the proton and neutron magnetic moments, g_A/g_V ratio and of the Q^2-dependence of the proton magnetic form factor. The obtained parameters are

$$m = 220\,\text{Mev} , \quad \alpha = 375\ \text{Mev} . \tag{28}$$

Note that these parameters differ slightly from those of Ref [9] the fact which is connected with the neglecting of quark anomalous magnetic moments in the present paper. The proton and neutron magnetic moments and g_A/g_V ratio corresponding to the parameters (28) are:

$$\mu_p = 2.93\ \text{n.m.} , \quad \mu_n = -1.8\ \text{n.m.} , \quad g_A/g_V = 1.16 \quad . \tag{29}$$

The predictions for form factors obtained with parameters (28) are given in Figs.1,2. It is seen that all nucleon electromagnetic form factors agree well with experiment, our predictions for the magnetic form factors being very close to the phenomenological dipole fit of experimental data:

$$G_D(Q^2)=(1+Q^2/0.71)^{-2} \qquad (30)$$

The predictions for nucleon axial vector form factor reproduce well the dipole behaviour of this form factor by formula:

$$G_A(Q^2)=(1+Q^2/M_A^2)^{-2} \qquad (31)$$

with M_A=1.15 GeV,the fact which agrees well with experimental data: M_A=1.03±0.04 GeV, M_A=1.06±0.05 GeV[25]. So, as it was expected from our qualitative analysis axial vector form factor falls significantly more slowly with increasing Q^2 than nucleon magnetic form factors. Let us consider in more detailes the predictions for $\Delta\to N\gamma$ transition.

(1) The obtained predictions for amplitudes $A^P_{1/2}$ and $A^P_{3/2}$ are noticable larger than in the nonrelativistic approximation but still smaller than experimental data(see Table). It is interesting that the relativistic corrections of order v^2/c^2 considered in Ref.[3] give similar result enlarging the magnitude of the amplitude $A^P_{3/2}$ up to 195 10^{-3}GeV $^{-1/2}$. Let us note that for nucleon and Δ-isobar considered as pure members of multiplet [56,0^+] there is no possibility in our approach to obtain larger values for $\Delta\to N\gamma$ amplitudes because our formulae in this case relate $\tilde{G}_1(0)$, $G^\Delta_M(0)$,$G^\Delta_E(0)$ and $A^P_{3/2}$ to the neutron magnetic moment in the following way:

$$e\tilde{G}_1(0) = \frac{3}{4m_N} e\,[G^\Delta_M(0) + G^\Delta_E(0)]=-\sqrt{3}\,\mu_n \qquad (32.1)$$

$$A^P_{3/2} = \sqrt{q_{cms}\; 3m_\Delta/\,2m_N}\;\mu_n \qquad (32.2)$$

The amplitudes of $\Delta\to N\gamma$ transition given in Table correspond to the neutron magnetic moment (29) obtained with parameters (28).

(2) As was expected from above made qualitative analysis the magnetic form factor of $\Delta\to N\gamma$ transition falls with increasing Q^2 more rapidly than nucleon ones. The quantitative calculations conform this conclusion, but the predicted form factor is however a little higher than experimental data (Fig.2). This fact will permit to include admixtures of higher exitation states into nucleon and isobar, which will be considered later.

(3) We have obtained non-zero value for Coulombic form factor of $\Delta\to N\gamma$ transition (see Fig.2) due to the relativistic effects. The obtained predicition agrees well with the results of the multipole analysis of pion electroproduction data [24]. Let us note that relativistic effects considered in Ref.[7] give also non-zero value for $G^\Delta_C(Q^2)$ with the same sign as our results but of lower size.

(4) Due to the relativistic effects we have obtained also non-zero value for the ratio E_{1+}/M_{1+} which agrees with experiment (see Table). By order of magnitude and sign this prediction coincides with predictions of chiral bag models of Refs. [29,30]. Non-zero value for the ratio E_{1+}/M_{1+} of smaller size (-0.2%) is obtained also in Ref.[31] in the light-cone model which is close to our model but uses different principle of construction of spin-orbital parts of wave functions. Similar predictions for E_{1+}/M_{1+} are obtained in Refs.[7,32,33] from configuration mixings.

(5) We predict a non-trivial Q^2-dependence of electric form factor $G^\Delta_E(Q^2)$ as well of the ratio E_{1+}/M_{1+} which change signs at Q^2=0.4 GeV^2. This fact can be checked by measuring of the helicity asymmetry which in this case should be lower in magnitude

than 0.5 at $Q^2 > 0.4$ GeV^2. So, in our approach we predict an effect which is similar to the effect predicted by pertubative QCD: $E_{1+}/M_{1+} \to 1$ (hence, $A \to 1$) at $Q^2 \to \infty$ [34,35]. It is, however, reasonable to expect that the prediction of perturbative QCD will be realized at higher values of Q^2.

3.2 The role of the admixtures of higher exitation states

Let us consider at first the case, when QCD-inspired admixtures of higher exitation states in N and Δ are minimal as it is found in Ref.[36]:

$$b_N \approx c_N \approx -b_\Delta \approx -0.11 \quad . \tag{33}$$

Parameters m and α of our model obtained in this case from the requiremnt of a best description of the proton and neutron magnetic moments, g_A/g_V ratio and Q^2-dependence of proton magnetic form factor are

$$m=220 \text{ Mev} , \quad \alpha=350 \text{ Mev} . \tag{34}$$

The predictions for other quantities corresponding to these parameters and to higher exitation states with coefficients (33) practically coincide with the predictions in the above cosidered case when admixtures of higher configurations in N and Δ are absent. The only exception is the magnetic form factor of $\Delta \to N\gamma$ transition which falls now more rapidly in the agreement with the experiment. This fact has qualitative explanation. If we consider the ratio of squared radia of nucleon and Δ-isobar in the nonrelativistic approximation we will obtain

$$r_\Delta^2/r_N^2 \approx 1 + 2(b_\Delta - b_N)/\sqrt{3} \quad . \tag{35}$$

As for QCD-inspired mixings $b_\Delta - b_N > 0$, the admixture of radial exitations in N and Δ results in larger radius for Δ-isobar and, hence, in more rapid fall of $\Delta \to N\gamma$ form factor in comparison with those for $N \to N\gamma$ transition. The admixture of multiplet [70,0$^+$] in N does not affect the ratio (35). So, the more rapid fall of $\Delta \to N\gamma$ magnetic form factor in the case under consideration is connected with the admixtures of radial exitations in N and Δ with signs which follow from QCD.

With increasing of the admixtures of higher exitation states in N and Δ (according to the results of Refs.[17,32]) the agreement between theory and experiment for $\Delta \to N\gamma$ transition becomes worse: the magnetic form factor of this transition falls more rapidly with increasing Q^2 and amplitudes $A^P_{1/2}$, $A^P_{3/2}$ become smaller. So, the best results for the amplitudes of $\Delta \to N\gamma$ transition may be obtained if in addition to the relativistic effects we take into account not large QCD-inspired admixtures of the higher exitation states in N and Δ of a size obtained in Ref.[36].

4 Conclusion

In the framework of a relativistic quark model of Ref.[9] we have performed the joint analysis of $N \to N\gamma$ and $\Delta \to N\gamma$ transitions in order to investigate the role of relativistic effects and QCD-inspired mixings in the description of $\Delta \to N\gamma$ form factors. It is shown that relativistic effects improve the agreement between quark model predictions and experimental data for amplitudes $A^P_{1/2}$ and $A^P_{3/2}$, however neither relativistic effects nor higher states admixtures permit to obtain total agreement with experiment in the magnitudes of these amplitudes. Apparently, we need the inclusion of additional physical mechanisms to achieve the agreement with experiment here. One of such mechanisms connected with gluonic deegrees of freedom is considered in Ref.[37]. It gives a contribution in right direction, but the effect is too small and additional mechanisms are necessary.

The relativistic effects and not large admixtures of multiplet $[56',0^+]$ in the N and the Δ with signs given by QCD allow to describe well the Q^2 dependence of magnetic form factor of $\Delta \to N\gamma$ transition.

An interesting prediction for electric form factor of $\Delta \to N\gamma$ transition is obtained. This form factor, being positive at $Q^2 < 0.4\ \mathrm{GeV}^2$, changes its sign with increasing Q^2, so the helicity asymmetry should be in magnitude lower than 0.5 at $Q^2 > 0.4\ \mathrm{GeV}^2$. Although the similar effect is predicted in perturbative QCD [34,35] we think that these effects are not overlapping, because it is naturally to expect that the perturbative QCD prediction will be realized at significantly larger values of Q^2.

I would like to express my gratitude to Nathan Isgur who had drawn my attention to the consideration of QCD-inspired admixtures of higher exitation states in the N and the Δ.

Appendix

We will give here formulae for matrix elements (3) taken between concrete members of multiplets $[56,0^+]$, $[56',0^+]$ and $[70,0^+]$ with positive charge. Matrix elements for states with zero charge can be obtained by replacements $Q_u \to Q_d$, $Q_d \to Q_u$.

Let us write matrix elements (3) in the form:

$$\frac{1}{2P}\langle B(J',\lambda')|J_o|A(J,\lambda)\rangle\Big|_{P\to\infty} = eN_A N_B \int I^{JJ'}_{\lambda\lambda'} \cdot \exp\left[-(M_o+M_o'^{\,2})/6\alpha^2\right] d\Gamma \,, \tag{A1}$$

where J,J' - are spins of particles, and N_A, N_B are normalizing factors which are equal to

$$[56,0^+]:\quad N=\left[\int d\Gamma\ \exp(-M_o^2/3\alpha^2)\right]^{-1/2}, \tag{A2}$$

$$[56',0^+]:\quad N=\left[\int d\Gamma(\vec\rho^{\,2}+\vec\lambda^2-3\alpha^{-2})^2\exp(-M_o^2/3\alpha^2)\right]^{-1/2}, \tag{A3}$$

$$[70,0^+]:\quad N=\left\{\int d\Gamma\left[(\vec\rho^{\,2}-\vec\lambda^2)^2+4(\vec\rho\vec\lambda)^2\right]\exp(-M_o^2/3\alpha^2)\right\}^{-1/2}, \tag{A4}$$

$$\rho=(K_c-K_b)/\sqrt{2}\,,\quad \lambda=(K_b+K_c-2K_a)/\sqrt{6} \tag{A5}$$

The quantities $I^{JJ'}_{\lambda\lambda'}$ have the form:

$[56,0^+] \to [56,0^+]+\gamma$:

$$I^{1/2\ 1/2}_{1/2\pm1/2}=(Q_u/2+Q_d)\ t^{(\pm)}_{\lambda\lambda}+3/2\ Q_u t^{(\pm)}_{\rho\rho}\,. \tag{A6}$$

$$I^{3/2\ 1/2}_{3/2\pm1/2}=(Q_u-Q_d)\ t^{(\pm)}_{\lambda}\,. \tag{A7}$$

$[56',0^+] \to [56,0^+]+\gamma$

$$I^{1/2\ 1/2}_{1/2\pm1/2}=\left\{(Q_u/2+Q_d)\ t^{(\pm)}_{\lambda\lambda}+3/2\ Q_u\ t^{(\pm)}_{\rho\rho}\right](\vec\rho^{\,2}+\vec\lambda^2-3\alpha^{-2})\,. \tag{A8}$$

$$I^{3/2\ 1/2}_{3/2\pm1/2}=(Q_u-Q_d)\ t^{(\pm)}_{\lambda}(\vec\rho^{\,2}+\vec\lambda^2-3\alpha^{-2})\,. \tag{A9}$$

$[70,0^+] \to [56,0^+]+\gamma$:

$$\begin{aligned}I^{1/2\ 1/2}_{1/2\pm1/2}=&\left\{Q_u/2+Q_d\right)\left[t^{(\pm)}_{\lambda\lambda}(\vec\rho^{\,2}-\vec\lambda^2)-2t^{(\pm)}_{\lambda\rho}\,\vec\rho\vec\lambda\right]+\\&+3/2\ Q_u\left[t^{(\pm)}_{\rho\rho}(\vec\lambda^2-\vec\rho^{\,2})-2t^{(\pm)}_{\rho\lambda}\vec\rho\vec\lambda\right].\end{aligned} \tag{A10}$$

$$I^{3/2\ 1/2}_{1/2\pm1/2}=(Q_u-Q_d)\left[t^{(\pm)}_{\lambda}(\vec\rho^{\,2}-\vec\lambda^2)-2t^{(\pm)}_{\rho}\vec\rho\vec\lambda\right], \tag{A11}$$

$$\begin{aligned}t^{(\pm)}_{\lambda\lambda}=&\{t(112112)+t(112121)+t(121112)+t(121121)+\\&4t(211211)-2[t(112211)+t(121211)+t(211112)+t(211121)]\}/6,\end{aligned} \tag{A12}$$

$$t^{(\pm)}_{\rho\rho}=[t(112112)-t(112121)-t(121112)+t(121121)]/2, \tag{A13}$$

$$t^{(+)}_{\lambda\rho}=[t(112112)-t(112121)+t(121112)-t(121121)-$$

$$-2t(211112) + 2t(211121)]/2\sqrt{3}\ , \qquad (A14)$$

$$t^{(+)}_{\rho\lambda} = [t(112112) - t(121112) + t(112121) - t(121121) -$$

$$- 2t(112211) + 2t(121211)]/2\sqrt{3}\ , \qquad (A15)$$

$$t^{(+)}_{\lambda} = [t(112111) + t(121111) - 2t(211111)]/\sqrt{6}\ . \qquad (A16)$$

$$t^{(+)}_{\rho} = [t(112111) - t(121111)]/\sqrt{2}\ , \qquad (A17)$$

$$t(i\ j\ k\ i'j'k') \equiv a^{(a)}_{ii'}\ a^{(b)}_{jj'}\ a^{(c)}_{kk'}\ / D_a D_b D_c\ , \qquad (A18)$$

where $a^{(a)}_{ii'}$, $a^{(b)}_{jj'}$, $a^{(c)}_{kk'}$ are matrix elements of products of Melosh matrices for quarks a,b,c which are given in Eqs (13). The quantities $t^{(-)}$ can be obtained from $t^{(+)}$ by replacements $t(112112) \to -t(221112)$, $t(121112) \to -t(212112)$ and so on. Note, that all our sign conventions for wave functions coincide with those of Isqur et al [17].

Formulae for the axial-vector form factor (22) for transitions between various multiplets are following :

$$g_A(Q^2) = N_A N_B \int A \exp\left[-(M_0^2 + M_0'^2)/6\alpha^2 \right] d\Gamma\ , \qquad (A19)$$

where for $[56,0^+] \to [56,0^+]$

$$A = (-P_{\lambda\lambda} + 3P_{\rho\rho})/2\ . \qquad (A20)$$

for $[56',0^+] \to [56,0^+]$

$$A = (-P_{\lambda\lambda} + 3P_{\rho\rho})(\vec{\rho}^2 + \vec{\lambda}^2 - 3\alpha^{-2})/2, \qquad (A21)$$

for $[70,0^+] \to [56,0^+]$

$$A = [-(P_{\lambda\lambda} + 3P_{\rho\rho})(\vec{\rho}^2 - \vec{\lambda}^2) + 2(P_{\lambda\rho} - 3P_{\rho\lambda})\vec{\rho}\vec{\lambda}]/2\ . \qquad (A22)$$

The quantities $P_{\lambda\lambda}$, $P_{\rho\rho}$, $P_{\lambda\rho}$ and $P_{\rho\lambda}$ in (A19)-(A 22) can be obtained from (A12)-(A15) by replacement $a^{(a)}_{ii'} \to \alpha_{ii'}$.

In order to obtain the results of nonrelativistic quark model and to investigate qualitatively the Q^2-dependence of form factors let us give the expressions for $t^{(\pm)}$ and P in the nonrelativistic limit: $\zeta = 1/2$, $K_{i\perp} = 0$ (i=a,b,c) , conserving the terms $O(q_x^0)$ and $O(q_x)$:

$$t^{(+)}_{\lambda\lambda} = t^{(+)}_{\rho\rho} = 1\ ,\ t^{(+)}_{\lambda} = \frac{\sqrt{6}}{4m}\eta q_x\ ,\ t^{(+)}_{\rho} = 0, \qquad (A22)$$

$$t^{(-)}_{\lambda\lambda} = -t^{(-)}_{\rho\rho} = \eta q_x\ . \qquad (A23)$$

$$P_{\lambda\lambda} = -1/3\ ,\ P_{\rho\rho} = 1\ . \qquad (A24)$$

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Table

Amplitudes of $\Delta \to N\gamma$ transition

	$A^p_{1/2}(10^{-3}\ GeV^{-1/2})$	$A^p_{3/2}(10^{-3} GeV^{-1/2})$	$E_{1+}/M_{1+}(\%)$	$S_{1+}/M_{1+}(\%)$
Our predictions	-112	-209	-1.9	-2
nonrelativistic quark model	-101	-175	0	0
experiment	-138±4 -136±6 [26]	-259±6 -247±10 [26]	-1.88±0.22 [27] -1.5±0.2 [28]	

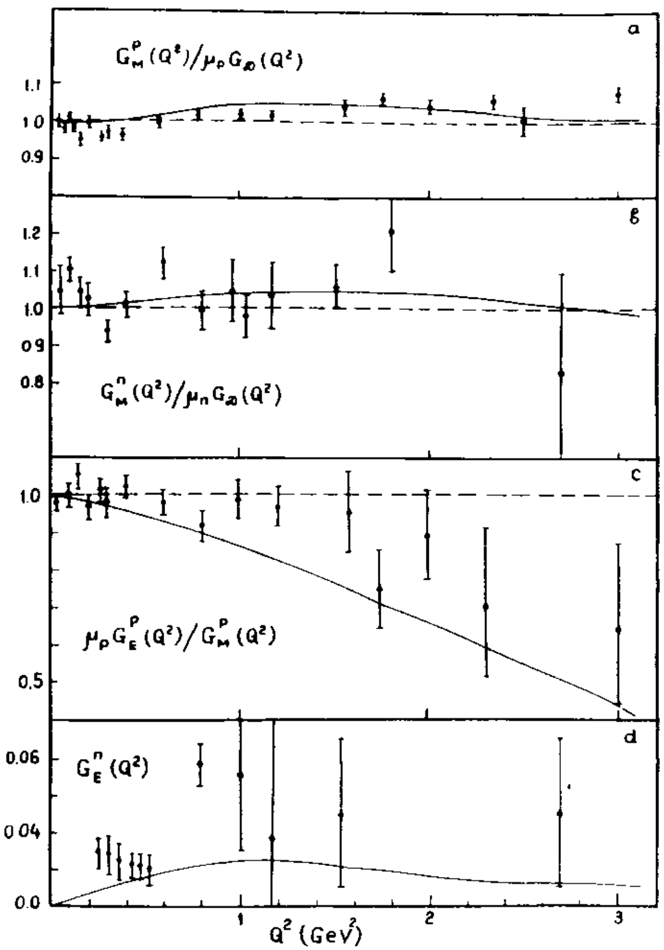

Fig. 1 The predictions for nucleon form factors. Data are taken form [22,23].

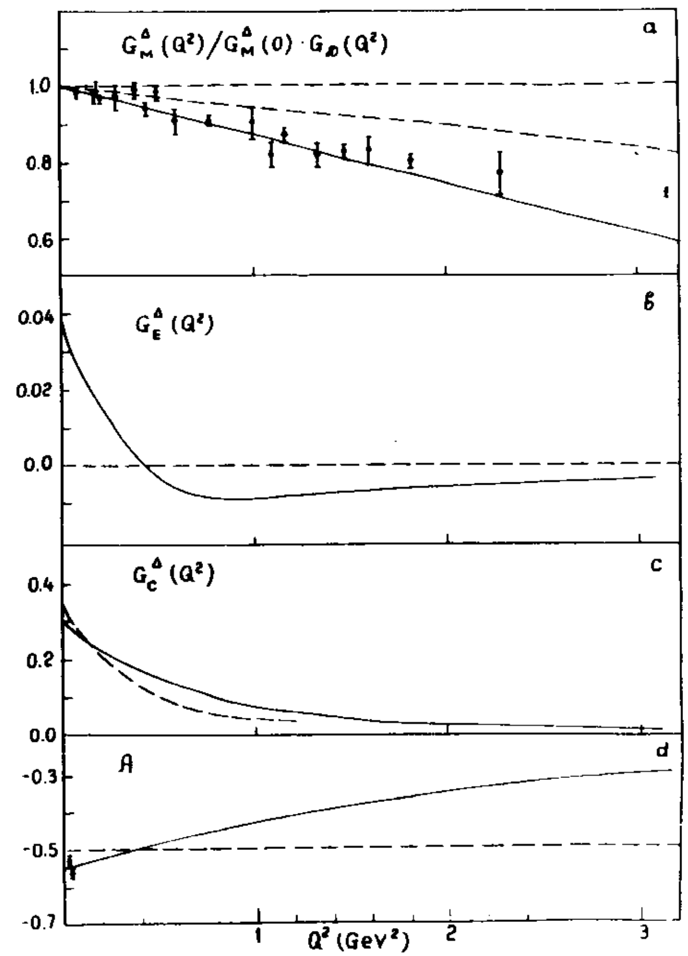

Fig 2 The predictions for $\Delta \to N\gamma$ transition. The dashed line on Fig. 2a is obtained taking into account only relativistic effects; the solid line corresponds to the inclusion of higher exitation admixtures in the N and the Δ with the coefficients (33); data are taken from [4,5]. The dashed line on Fig 2c correspond to the pion electroproduction multipole analysis [24]. On Fig 2d the prediction for helicity asymmetry $A = (A_{1/2}^2 - A_{3/2}^2)/(A_{1/2}^2 + A_{3/2}^2)$ is presented.

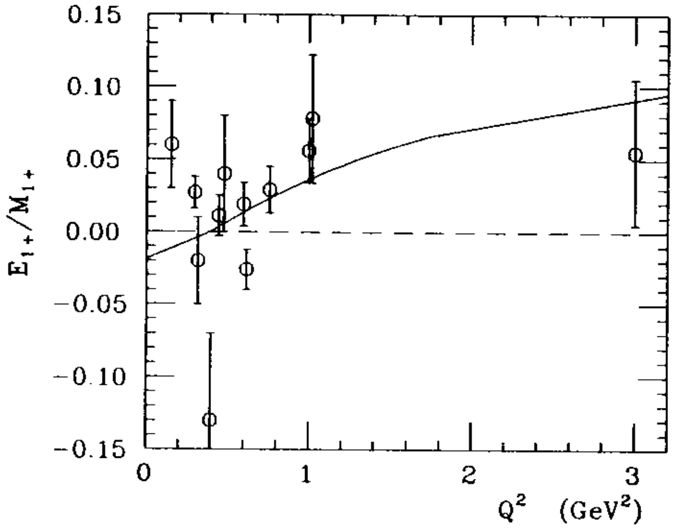

Fig.3 Our prediction for the ratio E_{1+}/M_{1+} and experimental data.